Approximate fibrations in higher topos theory

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Approximate fibrations, introduced by Coram and Duvall in [CPFD77], form a class of maps between topological spaces that extends the notion of proper Hurewicz fibrations. Informally, they are continuous functions satisfying a relaxed homotopy lifting property, in the sense that lifts exist only up to an appropriate notion of closeness. A notable family of approximate fibrations that are not necessarily Hurewicz fibrations is given by the so-called *cell-like maps* [Lac69]. Approximate fibrations and cell-like maps have been extensively studied and have numerous applications in geometric topology and manifold theory (see for example [Edw80]).

In this talk, we will explain how to recast the theory of approximate fibrations within the framework of higher topos theory. Using *internal higher category theory*, in the sense of Johnstone and, more recently, Martini–Wolf [MW25, MW23], we will introduce the notion of an approximate fibration between ∞-topoi. We will then sketch a proof that if $f : X \to Y$ is a proper continuous map between locally compact ANRs (e.g. locally finite CW-complexes), then f is an approximate fibration if and only if the associated geometric morphism $f_* : \mathcal{S}hv(X) \to \mathcal{S}hv(Y)$ is an approximate fibration. This generalizes an analogous theorem of Jacob Lurie, which provides a topos-theoretic characterization of cell-like maps [Lur09, Proposition 7.3.6.6]. Time permitting, we will discuss applications of our result to simple homotopy theory and state a conjecture regarding proper Hurewicz fibrations.

This is a joint work with Christian Kremer. You can find our preprint on the arxiv [KV25].

References

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