∞-Dold–Kan correspondence via representation theory

Chiara Sava

Let k be an algebraically closed field and Q a directed graph. To study the properties of the *path algebra* kQ, in representation theory we consider its *derived category* $\mathrm{D}(kQ)$, namely the category of chain complexes localized at quasi-isomorphisms. This category is, in particular, *triangulated*, meaning it is equipped with an additional structure that allows the computation of certain *homotopy limits and colimits* via mapping cones. However, this construction is not functorial. To address this issue, we instead work with *stable derivators* [Gro13]—enhancements of triangulated categories that provide additional structure enabling the definition of *homotopy Kan extensions*, and consequently, of homotopy limits and colimits.

Beyond the setting of representations over a field, little is known about the derived categories when considering more general coefficients. However, recently it has been observed that a lot of properties of categories of representations are actually mere consequences of the stability —in the sense of homotopy— of the categories involved and so they hold in a much broader generality. Along this line, in the first part of the talk, we will present a purely derivator-theoretic reformulation of a classical equivalence proved by Happel and Ladkani [Lad13], showing that it occurs uniformly across stable derivators and is therefore independent of coefficients.

Moreover, in [KN02], Keller and Neeman proved that a strong connection between representation theory and homotopy theory exists. Such connection was then further developed in the framework of derivators by Groth, Ponto, Shulman and Šťovíček in [GPS14] and [GŠ18]. Building on this perspective, in the second part of the talk, we will explain how the equivalence obtained in the first part can be viewed as a derivator-theoretic version of the ∞*-Dold–Kan correspondence* (see [Ari21] and [Lur11]) for bounded chain complexes, thereby providing a bridge between homotopy theory and representation theory. Indeed, the Dold–Kan correspondence is a central result in homotopy theory, underlying, for example, the definition of singular homology.

This talk is based on the arXiv preprint [Sav22].

References

[Ari21] Stefano Ariotta. *Coherent cochain complexes and Beilinson t-structures, with an appendix by Achim Krause.* arXiv:2109.01017, 2021.

[Gro13] Moritz Groth. *Derivators, pointed derivators, and stable derivators.* Algebr. Geom. Topol., 13:313–374, 2013.

[GPS14] Moritz Groth, Kate Ponto, and Michael Shulman. *The additivity of traces in monoidal derivators.* J. K-Theory, 14(3):422–494, 2014.

[GŠ18] Moritz Groth and Jan Šťovíček. *Tilting theory via stable homotopy theory.* J. reine angew. Math. 743 (2018), 29-90, doi:10.1515/crelle-2015-0092

[KN02] Bernhard Keller and Amnon Neeman. *The connection between May's axioms for a triangulated tensor product and Happel's description of the derived category of the quiver D4.* Doc. Math., 7:535–560 (electronic), 2002.

[Lad13] Sefi Ladkani. *On derived equivalences of lines, rectangles and triangles.* J. Lond. Math. Soc. (2) 87 (2013), no. 1, 157-176.

[Lur11] Jacob Lurie. *Higher algebra.* Preprint, 2011.

[Sav22] Chiara Sava. *∞-Dold-Kan correspondence via representation theory.* (2022) preprint at https://arxiv.org/abs/2211.00762v3