Towards a notion of coherent and ideal actions in ideally exact contexts

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Abstract.

In the context of ideally exact categories, we introduce the notions of internal *coherent* action and internal *ideal* action that generalize different aspects of unital actions of rings. We prove that every ideal action is coherent, and we call *BAT*[1] the ideally exact contexts with a *good theory of actions*, i.e., where all coherent actions are ideal and all morphisms of such actions are ideal. Eventually, we present some case studies of BAT contexts: unital non-associative $\mathbb{F}$-algebras and ring, MV-algebras, product algebras, and $\mathsf{Set}^{\mathrm{op}}$, the dual of the category of sets.

This is joint work with Manuel Mancini (*Università degli Studi di Palermo*) and Giuseppe Metere (*Università degli Studi di Milano Statale*).

References

[1] M. Mancini, G. Metere and F. Piazza, *Coherent and ideal actions in ideally exact categories*, submitted, preprint arXiv:2507.06124.

[1] The acronym BAT is inspired by the notion of BIT-variety, where BIT stands for **B**uona (good, in Italian) **I**deal **T**heory, introduced by A. Ursini. Analogously, BAT stands for **B**uona **A**ction **T**heory.