Semi-strictification of (∞, n)-categories

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Abstract.

We prove the first equivalence between a weak non-algebraic model and a semi-strict algebraic model of (∞, n)-categories. This takes the form of a natural semi-strictification, whereby a weak (∞, n)-category is embedded into a semi-strict one through an acyclic cofibration, in such a way that weak functors lift to semi-strict functors; this constitutes the derived unit of a Quillen equivalence between weak model categories whose fibrant objects are, respectively, the weak (∞, n)-categories and (up to an acyclic fibration) the semi-strict ones. The semi-strict model has algebraic units and composition of round pasting diagrams, satisfying a strict form of associativity and interchange as in Henry's regular version of Simpson's weak units conjecture; semi-strict functors strictly preserve round composition, but only weakly preserve units. Globular composition operations are obtained from a combination of units and round composition. Since the models satisfy the homotopy hypothesis in the case $n = 0$, this result also exhibits the first semi-strict model of the classical homotopy types that has algebraic units and composition. The constructions are based on the combinatorics of regular directed complexes and are entirely explicit and combinatorial, in the spirit of Mac Lane's strictification of bicategories.

References

[1] C. Chanavat and A. Hadzihasanovic. Semi-strictification of (∞, n)-categories. Online preprint arXiv:2507.00146, 2025.