Fibrations with comprehension and their completions

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Dependent types differ from simple types in many aspects. One of the most important difference concerns how they handle contexts. In particular many categorical structures have been proposed to model context extension. Here, we focus on two such structures: Jacobs' *comprehension categories* [3, 4] and Ehrhard's *D-categories* [2]. One of our goals consists in understanding the analogies and the differences between them.

A comprehension category is a pair (p, χ) of a fibration $p\colon \mathcal{E} \to \mathcal{B}$ and a so-called *comprehension functor* $\chi\colon \mathcal{E} \to B^2$, which has to preserve cartesian morphisms. This results in the context extension being well-behaved with respect to substitutions.

D-categories are a generalization of Lawvere's *hyperdoctrines satisfying the comprehension schema* [5], who introduced them in his attempt to express categorically the set theoretic axiom schema. A D-category is a fibration $p\colon \mathcal{E} \to \mathcal{B}$ with fibered terminal objects such that the functor $\mathrm{T}\colon \mathcal{B} \to \mathcal{E}$ that picks the terminals of the fibers has a right adjoint C.

In [3] Jacobs shows that comprehension categories and D-categories are strictly related, assigning (functorially) a comprehension category to a given D-category.

Due to the lack of strictness in the notions of comprehension category and of D-category, it appears natural to study these in a 2-categorical framework [1]. Working within it, we present a construction of the free comprehension category over a fibration, and of the free D-category over a fibration with fibered products. In the end, we characterize which comprehension categories arise from a D-category through Jacobs' construction.

References

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