ON THE ESSENTIALLY ALGEBRAIC 2-THEORY GENERATED BY A TYPE THEORY

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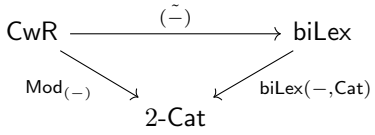
Building up on the main idea of Awodey's Natural models ([Awo18]), Uemura introduced in [Uem23] a functorial semantics for Dependent Type Theory through his notion of categories with representable maps (CwR).

This work in progress wants to push forward the study of the 2-category of models of a given CwR while clarifying the position of Uemura's theory within the broader landscape of functorial semantics. Following [Pow95]'s motto that every fundamental structure arising in computer science is the model of an essentially algebraic 2-theory, we show that to every CwR $\mathbb{T}$ one can functorially associate a finitely bilimit-complete 2-category $\tilde{\mathbb{T}}$ such that the 2-category of bilex pseudofunctors $\tilde{\mathbb{T}} \to \mathsf{Cat}$ coincides with that of models of $\mathbb{T}$ as in the diagram:



References

[Awo18] Steve Awodey. Natural models of homotopy type theory. *Mathematical Structures in Computer Science*, 28(2):241–286, 2018.

[Pow95] John Power. Why tricategories? *Information and Computation*, 120(2):251–262, 1995.

[Uem23] Taichi Uemura. A general framework for the semantics of type theory. *Mathematical Structures in Computer Science*, 33(3):134–179, 2023.

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