

title: Presheaves, Sheaves and Sheafification via triposes

abstract:

The notion of tripos was originally introduced by Hyland, Johnstone and Pitts to explain from an abstract perspective in which sense localic sheaf toposes and Hyland's realizability toposes are instances of the same construction.

The main purpose of this work is to further investigate the common structures of these classes of toposes from a more geometric point of view.

In particular, we first introduce an exact category of "abstract presheaves" for (arbitrary-based) triposes by combining the tripos-to-topos construction with the full existential completion. The given name is motivated by the fact that abstract presheaves coincide with localic presheaves in the case of localic triposes. Then, we call  $\exists$ -sheaf triposes those triposes whose abstract presheaves category is a topos, and we prove that every Set-based tripos is a  $\exists$ -sheaf tripos.

Furthermore, we show that the sheafification between a localic topos and its presheaf topos can be generalized to an "abstract sheafification adjunction" between a  $\exists$ -sheaf triposes and its full existential completion.

In particular, we conclude that any tripos-to-topos construction of a Set-based tripos can be seen as the category of  $j$ -sheaves for the Lawvere-Tierney topology  $j$  induced by an abstract sheafification adjunction.

This talk is based on joint work with Maria Emilia Maietti.