A Stone duality for the class of compact T_1 spaces

Elena Pozzan

Università di Torino, Torino, Italy
elena.pozzan@unito.it

Abstract

It is well known that we can characterize T_0-topological spaces in terms of preorders describing a base for the space. In particular, any T_0-topological space can be represented as the space whose points are the neighborhood filters of one of its basis for the open sets. Conversely, we show that any dense family of filters on a preorder defines a topological space whose characteristics are strictly connected to the ones of the preorder. Therefore, we show how the separation properties of the topological space can be described in terms of the algebraic properties of the corresponding preorder and family of filters.
Furthermore, we outline the algebraic conditions on a selected base of the topological space ensuring that the space is compact and T_1. This allows us to establish a duality between the category of compact T_1 spaces with continuous closed maps and an appropriate category of lattices. Moreover, we could specialize this duality to the category of compact Hausdorff spaces with continuous maps.
Weakening these results, we will also present two contravariant adjunctions between these categories of topological spaces and some category of elementary lattices that are first-order describable.
These characterizations allow us to give a description of the Stone–Čech compactification of a topological space in terms of lattices.
This is joint work with Matteo Viale.

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