Quasi-Hopf algebras of dimension 6

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The classification of quasi-Hopf algebras is in an early age, with most advancements focusing on the semisimple or basic case. With respect to the semisimple case, this is due to the fact that semisimple quasi-Hopf algebras characterize fusion categories with integer Frobenius-Perron dimensions of simple objects. Thus, the classification of such fusion categories provides for free the classification of the semisimple quasi-Hopf algebras and vice versa. For instance, this is the case in dimension p and pq, see [4] and [3], respectively, provided that p < q are positive prime integers.

We completed the classification of the 6-dimensional quasi-Hopf algebras, by proving that any such algebra is semisimple; this is achived by classifying braided Hopf algebras in left Yetter-Drinfeld module categories over a quasi-Hopf algebra, which are categorically equivalent to biproduct quasi-Hopf algebras in the sense of Radford [2]. As byproducts, we provide examples of 6-dimensional quasi-bialgebras that are not semisimple as algebras, as well as the concrete quasi-Hopf structures of the 6-dimensional semisimple quasi-Hopf algebras previously classified by Etingof and Gelaki in terms of their category of representations in [3]. (Based on a joint work with Daniel Bulacu)

References

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