DIFFERENTIAL AUTOMATA THEORY

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We study generalized automata (in the sense of Adámek-Trnková) in Joyal's category of (set-valued) combinatorial species, and as an important preliminary step, we study coalgebras for its derivative endofunctor ∂ and for the 'Euler homogeneity operator' $L \circ \partial$ arising from the adjunction $L \dashv \partial \dashv R$.

The theory is connected with, and in fact provides relatively nontrivial examples of, *differential 2-rigs*, a notion recently introduced by the author putting combinatorial species on the same relation a generic (differential) semiring (R, d) has with the (differential) semiring $\mathbb{N}[\![X]\!]$ of power series with natural coefficients.

The desire to study categories of 'state machines' valued in an ambient monoidal category $(\mathcal{K}, \otimes)$ gives a pretext to further develop the abstract theory of differential 2-rigs, proving lifting theorems of a differential 2-rig structure from $(\mathcal{R}, \partial)$ to the category of ∂-algebras on objects of $\mathcal{R}$, and to categories of Mealy automata valued in $(\mathcal{R}, \otimes)$, as well as various constructions inspired by differential algebra such as jet spaces and modules of differential operators.

These theorems adapt to various 'species-like' categories such as coloured species, k-vector species (both used in operad theory), linear species (introduced by Leroux to study combinatorial differential equations), Möbius species, and others.

This talk covers the content of [Loregian, 2024], as well as parts of an ongoing project with Todd Trimble.

References

[Loregian, 2024] Loregian, F. (2024). *Automata and Coalgebras in Categories of Species*, page 65–92. Springer Nature Switzerland.

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