A 2-categorical analysis of context comprehension

Greta Coraglia[1] and Jacopo Emmenegger[2]

[1] Dipartimento di Filosofia, Dipartimento di Eccellenza 2023-2027, Università di Milano
greta.coraglia@unimi.it

[2] Dipartimento di Matematica, Dipartimento di Eccellenza 2023-2027, Università di Genova
jacopo.emmenegger@unige.it

The problem of modelling the structural rules of type dependency using categories has motivated the study of several structures, varying in generality, occurrence in nature, and adherence to the syntax of dependent type theory. One aspect, that involving free variables and substitution, is neatly dealt with using (possibly refinements of) Grothendieck fibrations. The other main aspect of type dependency is the possibility of making assumptions as encoded in the two rules below

$$\frac{\Gamma \vdash A\ \texttt{Type}}{\vdash \Gamma.A\ \texttt{ctx}} \qquad\qquad \frac{\Gamma \vdash A\ \texttt{Type}}{\Gamma.A \vdash \mathtt{v}_A : A}$$

where the first one (*context extension*) extends the context Γ with the type A, and the second one (*assumption*) provides a "generic term" of A in context $\Gamma.A$. In the first order setting, they allow us to add assumptions to a context, and to prove what has been assumed, respectively.

We present a purely 2-categorical comparison of the two main categorical accounts of these two rules: Jacobs' comprehension categories [Jac99] and Dybjer's categories with families [Dyb96]. They differ in that the former gives prominence to context extension, and the latter to assumption. The comparison itself consists of a biequivalence of 2-categories, which generalises the classical 1-equivalence between the discrete versions of these structures due to Hofmann [Hof97]. The biequivalence itself is obtained from the (co)Structure-Semantics Adjunction [Dub70, Str72]. These results have just been published in TAC [CE24].

References

[CE24] Greta Coraglia and Jacopo Emmenegger. A 2-categorical analysis of context comprehension. *Theory and Applications of Categories*, 41(42):1476-1512, 2018.

[Dub70] Eduardo Dubuc. *Kan Extensions in Enriched Category Theory*, volume 145 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin-New York, 1970.

[Dyb96] Peter Dybjer. Internal type theory. In Stefano Berardi and Mario Coppo, editors, *Types for Proofs and Programs*, pages 120–134, Berlin, Heidelberg, 1996. Springer Berlin Heidelberg.

[Hof97] Martin Hofmann. Syntax and semantics of dependent types. In Andrew M. Pitts and Peter Dybjer, editors, *Semantics and Logics of Computation*, Publications of the Newton Institute, page 79–130. Cambridge University Press, 1997.

[Jac99] Bart Jacobs. *Categorical logic and type theory*. Elsevier, 1999.

[Str72] Ross Street. The formal theory of monads. *Journal of Pure and Applied Algebra*, 2(2):149–168, 1972.