On continuity of functors between locally presentable categories

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We prove that for each locally finitely presentable category $\mathcal{K}$ there exists a regular cardinal κ such that any finitary functor out of $\mathcal{K}$ (into another locally finitely presentable category) is continuous if and only if it preserves κ-small limits; as a consequence we obtain a new adjoint functor theorem specific to the finitary functors out of $\mathcal{K}$.

While this is all very nice (and maybe unexpected), one might wonder whether there is any application of this result. I will present two: (1) this theorem allows us to characterize the dualizable modules over a commutative ring R using certain flatness conditions, and (2), generalizing this to the enriched setting, we deduce that a small $\mathcal{V}$-category is accessible if and only if it is Cauchy complete.