An unbiased approach to symmetry

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When we consider a symmetric and associative operation, it is often clear that we are in fact concerned with a function on *families*, rather than on sequences, of elements. The spurious order, while it may appear to have a simplifying role, eventually results in unnecessary complications, as it often happens whenever a category (here the category of finite sets) is forced to be skeletal. The languages of double categories and of fibrations provide a natural setting to deal with those categorical notions which involve operations on families of objects, such as symmetric multicategories, symmetric monoidal categories, props and algebraic theories.

The idea is that, for a suitable base double category $\mathbb{B}$, a double category over it with some fibration conditions, $F : \mathbb{D} \to \mathbb{B}$, gives a general notion of symmetric operation. The base $\mathbb{B}$ allows for an explicit consideration of indexes with their transformations, while the functor F gives the indexing of objects and operations (that is, "loose" morphisms) in $\mathbb{D}$ and, through the fibration conditions, also their reindexing along arrows in $\mathbb{B}_1$.

In particular we show how symmetric multicategories (colored operads), in their non-skeletal or unbiased form, can be defined simply as double functors to the double category of pullback squares in finite sets

$$F : \mathbb{D} \to \mathbb{P}\mathrm{b}\,\mathbf{Set_f}$$

such that both the components F_0 and F_1 are sum-preserving discrete fibrations. Unbiased symmetric monoidal categories arise when the loose part of F is an opfibration.

We discuss the advantages of this approach and some possible developments.

References

[1] C. Pisani, Operads as double functors, arXiv (2022).