Indexed Grothendieck construction

The Grothendieck construction is a fundamental tool in category theory that regulates the equivalence between indexed categories and Grothendieck fibrations.

We present an indexed version of the Grothendieck construction. This gives a pseudonatural equivalence of categories between opfibrations over a fixed base in the 2-category of 2-copresheaves and 2-copresheaves on the Grothendieck construction of the fixed base. Such equivalence also restricts to discrete opfibrations and copresheaves.

We can think of the indexed Grothendieck construction as a simultaneous Grothendieck construction on every index that takes into account all bonds between different indexes. This will be clear in the explicit description of the indexed Grothendieck construction.

Our result is a 2-dimensional generalization of the equivalence between slices of copresheaves and copresheaves on slices. Thus it generalizes to dimension 2 the fundamental fact that every slice of a Grothendieck topos is again a Grothendieck topos.

As an application, we show that the indexed Grothendieck construction can be used to get a nice candidate for a Hofmann-Streicher universe in 2-presheaves.

This is a joint work with Elena Caviglia.