Weak representability of actions of non-associative algebras

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It is well known that, in the semi-abelian category **Lie** of Lie algebras over a field $\mathbb{F}$, algebra actions are represented by derivations. This means that the category **Lie** is *action representable* and the representing object, which is called the *actor*, is the Lie algebra of derivations. The notion of action representable category has proven to be quite restrictive. For example, if a non-abelian variety $\mathcal{V}$ of non-associative algebras over an infinite field $\mathbb{F}$, with $\mathrm{char}(\mathbb{F}) \neq 2$, is action representable, then $\mathcal{V}$ must be the category **Lie**. More recently G. Janelidze introduced the notion of *weakly action representable category*, which includes a wider class of categories, such as the variety **Assoc** of associative algebras and the variety **Leib** of Leibniz algebras.

In this talk we show that the converse of the implication

$$\textit{Weakly Action Representable Category} \Rightarrow \textit{Action Accessible Category}$$

is false also in the context of varieties of non-associative algebras. Then, for an *algebraically coherent* and *operadic* variety $\mathcal{V}$ and an object X of $\mathcal{V}$, we show that it is always possible to construct a *partial algebra* $\mathcal{E}(X)$, called *external weak actor* of X, and a monomorphism of functors

$$\tau\colon \mathrm{Act}(-,X) \rightarrowtail \mathrm{Hom}_{\mathbf{PAlg}}(-,\mathcal{E}(X)),$$

where **PAlg** is the category of partial algebras over $\mathbb{F}$. The pair $(\mathcal{E}(X),\tau)$ is called *external weak representation* of the functor $\mathrm{Act}(-,X)$. Moreover, for any other object B in $\mathcal{V}$, we provide a complete description of the morphisms $(B \to \mathcal{E}(X)) \in \mathrm{Im}(\tau_B)$, i.e. of the homomorphisms of partial algebras which identify the actions of B on X in $\mathcal{V}$.

This is joint work with Xabier García Martínez (*Universidade de Vigo*, Spain), Tim Van der Linden and Corentin Vienne (*Université catholique de Louvain*, Belgium).

References

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