Involutive Markov categories and the quantum de Finetti theorem

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Categorical probability, and in particular Markov categories, has attracted much interest in recent years. This synthetic approach has proven fruitful in several areas, such as statistics [1], graphical models [2], ergodic theory [3], and more. Regarding the quantum perspective, Parzygnat introduced the concept of quantum Markov categories in [4], where he focused on finite-dimensional C*-algebras. His work explains that, in the quantum perspective, probability is encoded by well-behaved subcategories rather than by the whole quantum Markov categories.

This talk offers an alternative description of quantum Markov Categories, which we call *involutive Markov categories*, and their subcategories of interest, which we call *pictures*. This view has the advantage of avoiding any distinction between odd and even morphisms. We then discuss a particular example which comprises infinite-dimensional (pre)-C*-algebras, and argue that the theory of such algebras can benefit from this new synthetic perspective.

Indeed, we are able to provide a meaningful framework for the quantum de Finetti theorem via *representability*. This concept is also important for classical Markov categories [1]. In particular, representability implies that the category under consideration is a Kleisli category [1, Theorem 3.19]. It turns out that a quantum version of representability can also arise from a well-behaved limit of exchangeable morphisms, which gives rise to a *de Finetti object*. We will prove that such objects exist, and thus a quantum de Finetti theorem holds in this setting.

References

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