

## 2-DIMENSIONAL ASPECTS OF THE THEORY OF SKETCHES

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The notion of *sketch* was introduced by Ehresmann [2, 3]. It consists of a category together with a specification of certain cones and cocones. Using the idea that certain logical operations can be described through limits and colimits, sketches have been considered as one of the many formalisation of the concept of *theory* [1, 4, 5]. In particular, they can be used to present theories in infinitary logics.

The aim of this work is to lay the foundations for applying 2-dimensional techniques in the study of sketches, having in mind applications in infinitary logics. In particular, we extend what was done with sites and topoi in the context of geometric logic to infinitary logic introducing the notions of *rounded sketch* and *logos*.

Logic Fragment	Presentation	Morita Classifying Object
Geometric	Site	Topos
Infinitary	Rounded Sketch	Logos

We start by showing some nice properties of the 2-category of sketches. For instance, we give an explicit formula to calculate weighted pseudo co/limits, taking advantage of a *topological behaviour*.

Then, we provide some normalisation constructions which will be useful for our main result, a Diaconescu-like theorem for rounded sketches and logoi. More precisely, the aim is to assign to any rounded sketch a *classifying logos*, following the idea that rounded sketches should represent presentations of theories (in some infinitary logic) and logoi the syntax-independent objects.

### REFERENCES

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