Relation transformers
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The notions of relation transformer and their morphisms generalize those of binary relation and morphism of relations to functors and natural transformations. It is then possible to define the notions of relation transformer graph $\langle \alpha \rangle$ of a natural transformation $\alpha \colon F \to G$ and, consequently, that of equality relation transformer Eq_F of a functor F.

As the main result, we show that, applying the trace of a natural transformation $\alpha \colon F \to G$ to the trace of a morphism $f \colon A \to B$, we get the trace of the diagonal given by the naturality square:

$$\langle \alpha \rangle(\langle f \rangle) = \langle G(f) \circ \alpha_A \rangle = \langle \alpha_B \circ F(f) \rangle$$

It follows as a corollary that $\mathrm{Eq}_F(\mathrm{Eq}_X) = \mathrm{Eq}_{F(X)}$.

This work has been published in [1].

References

[1] Patricia Johann and Enrico Ghiorzi. "Parametricity for Nested Types and GADTs." In: *Logical Methods in Computer Science* Volume 17, Issue 4 (Dec. 2021). DOI: 10.46298/lmcs-17(4:23)2021. URL: https://lmcs.episciences.org/8889.