

1 ItaCa 2023: Enriched ∞ -categories

Enriched category theory has an important role in homotopy and (co)homology theories: homotopy (co)limits are usually computed using Set_{Δ} -enriched categories, derived categories (which are triangulated) are usually studied using $Ch(k)$ -enriched categories, and so on. Nowadays, the theory of enriched ∞ -categories is under construction thanks to the work of Haugseng, Gepner, Lurie, Hinich, Heine and Macpherson. Since the theory of ∞ -categories brought new points of view of problems in homotopy and (co)homology theories, it is to be expected that the enriched ∞ -categories will be a further step in understanding these problems.

In the first talk I will give a short introduction about enriched ∞ -categories theory, pointing out what is missing to make it truly the higher version of the classic one.

In the second part of the talk I will prove that, given an \mathbb{E}_{∞} -ring \mathcal{A} , the ∞ -category of $LMod_{\mathcal{A}}$ -enriched ∞ -categories, $Cat_{\infty}^{LMod_{\mathcal{A}}}$, is equivalent to the ∞ -category of left \mathcal{A} -module objects of Cat_{∞}^{Sp} , $LMod_{\mathcal{A}}(Cat_{\infty}^{Sp})$. As far as I know, this is the first time that this result is clearly stated. This result allows to reduce (co)homology problems with general coefficients to integral (co)homology problems, which, in turn, can sometimes be approached using ∞ -category theory.