

The Exodromy Theorem

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Abstract

Let X be a connected topological space. By a classical theorem in algebraic topology, the category of locally constant sheaves on X with values in \mathbf{Set} is equivalent to the category of representations of $\pi_1(X)$. In particular, our category is of the form $\mathbf{Fun}(\mathcal{A}(X), \mathcal{B})$, where \mathcal{B} is a category which does not depend on X and is presentable. This presentation has interesting abstract categorical consequences (for example, the category of locally constant sheaves is itself presentable). This kind of presentation can be extended to other categories of sheaves, e.g. the category of constructible sheaves on a stratified space. This latter statement is due to Robert Macpherson and has been generalized to the setting of derived categories by Jacob Lurie. We explain the statement of the theorem and mention an application to our research.

Prerequisites

Basic algebraic topology (homotopy groups), basic category theory (presentable categories, symmetric monoidal categories). Lurie's theorem will only be mentioned briefly at the end, and will be accessible to those with basic knowledge in higher category theory and homological algebra.