

# Categorical Semantics for Counterpart-based Temporal Logics in Agda\*

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Temporal logics have proven to be one of the most effective techniques for the verification of both large-scale and stand-alone programs, and several formalisms have been developed in order to express the evolution of complex multi-component systems. One prominent example is the case of graph logics [1], where states are specialized as graphs and transitions are families of (partial) graph morphisms, thus capturing the evolution a graph topology over time. In order to express the creation, merging and deallocation of components, the *counterpart paradigm* introduced by Lewis [2] has been proposed as a framework in which the models of such multi-component logics can be provided: the idea is to identify a local set of elements for each state of a transition system, and to use possibly partial morphisms as transitions to carry the identity of elements from one state to the other. This perspective is applied in [3] to model a counterpart-based  $\mu$ -calculus with second order quantifiers, and it is generalized to the categorical setting in [4] using *relational presheaves* by building on the ideas of [5].

In this work we present the categorical semantics of a first-order temporal logic QLTL based on the counterpart paradigm. The perspective used in our work is to consider the objects of a category  $\mathcal{W}$  as worlds in a given instant of time, and to view morphisms as possible temporal evolutions. Then, a *counterpart  $\mathcal{W}$ -model* consists of a category  $\mathcal{W}$  along with a so-called *relational presheaf*  $D : \mathcal{W}^{op} \rightarrow \mathbf{Rel}$ , where  $\mathbf{Rel}$  is the category of sets and relations: such a presheaf assigns to each world a set of individuals, and to each morphism a corresponding relation between worlds. Following the counterpart paradigm, the individuals of two worlds are related by a relation  $R$  when they are the same individual after a temporal evolution. These models can also be generalized to express the temporal evolution of multi-sorted algebras on any signature  $\Sigma$  (e.g.: the signature of directed graphs) by providing relational presheaves for each sort of the signature along with adequate morphisms between them.

We also illustrate a computer-assisted formalization of these constructions and the categorical semantics using the proof assistant Agda, highlighting the crucial aspects of our formalization and the practical use of (quantified) temporal logics in a constructive proof assistant. Moreover, we present some results on the positive normal form of QLTL and discuss the related issues given by the *constructive* setting in which we embed our temporal logic. We employ the [agda-categories](#) library to adequately capture the notions of category theory in our setting.

## References

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